Realistic models of beliefs in a paraconsistent and paracomplete setting

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Evolution of Group Knowledge

Traditional Approaches		The New Approach
Simple Fusion "What every fool knows"	$\Longrightarrow$	**Contextual fusion** Synthetic information extracted from individuals or other groups
Holistic Knowledge Complex	$\Longrightarrow$	**Selected aspects** Efficient
Consensus Permanent, requiring belief change: update, merging or revisions	$\Longrightarrow$	**Shadowing** Agents not forced to adopt group beliefs: only required to obey them during the group's lifetime

Evolution of Group Knowledge

Traditional Approaches		The New Approach
Omniscience		**Commonsense**
All consequences known: hardly acceptable	$\Longrightarrow$	Incomplete/inconsistent beliefs allowed: information may be resolved non-monotonically
Homogeneity		**Heterogeneity**
Reasoning typically based on uniform principles	$\Longrightarrow$	Reasoning easily adjustable to application domain and individualized
Complexity		**Complexity**
Beyond P	$\Longrightarrow$	Tractable

Establishing Group Beliefs

Distributed Beliefs

"What a wise person would know":

pulls together the individual beliefs and draws classical conclusions from the combined information.

Group Beliefs

May go much further: from the same beliefs, a variety of reasoning or non-deductive methods (possibly originating from social choice) may lead to more far-reaching conclusions.

This takes place at our approach.

Important

Reducing complexity of communication and reasoning is essential (especially in time-critical situations).

Real-world Applications: Partiality

Partiality

When a belief base neither entails A nor $\neg A$.

Potential Sources

- General ignorance.
- Dynamic and unpredictable environments.
- Perception and imprecise measurements.
- Exploring unknown areas,
  e.g., after a catastrophe.

Real-world Applications: Inconsistencies

Inconsistencies

When a belief base entails both A and $\neg A$.

Appear at many different levels: individual, between agents, between agents and groups, between groups and groups.

Potential Sources

- Fusing beliefs from heterogeneous sources.
- Contradictory information as useful evidence.
- Commonsense/informal reasoning.
- Inconsistencies in models.
- Real-world situations,
  e.g., red light while a policeman directs to go through.

Shift in Perspective

Entailment

- General: applicable to arbitrary formulas.
- Complex.
- Often unnecessary.
- Theory-oriented.

$\Longrightarrow$

Querying

- Restricted to rules (price to be paid).
- Efficient.
- Needed.
- Database-oriented.

Shift in Perspective

Classical, Modal

- No tools for nonmonotonicity.
- Poor tools for belief fusion.
- Homogeneity of information sources.
- Inconsistencies trivialize reasoning.
- ...

$\Longrightarrow$

Paraconsistent, Modular

- Support for most forms of nonmonotonic reasoning.
- Support for belief fusion.
- Heterogeneity of information sources.
- Inconsistencies supported at many levels.
- ...

Belief Bases (Incomplete, Inconsistent Information, 3ı)

3i-world

- A finite set of ground literals.
- E.g., $\{safe(r1), \neg safe(r2), safe(r3), \neg safe(r3)\}$.

Truth Values

$safe(r1) = \boldsymbol{t}$, $safe(r2) = \boldsymbol{f}$, $safe(r3) = \boldsymbol{i}$, $safe(r4) = \boldsymbol{u}$.

Belief Base

- A finite set of $3i$-worlds.
- Rigid and flexible constraints.

Interpretations of 3i-worlds

- Alternative views.
- Complementary views (potentially fused).

More Formally

Let $\mathbb{C} \stackrel{\text{def}}{=} \mathrm{Fin}(\mathit{Const})$ be the set of all finite sets of ground literals over a set of constants *Const*.

- By a *3ı-world* we understand any set $C \in \mathbb{C}$.
- By a *belief base* we understand any finite set of 3ı-worlds.

Belief Structures [DKS]

Epistemic Profile

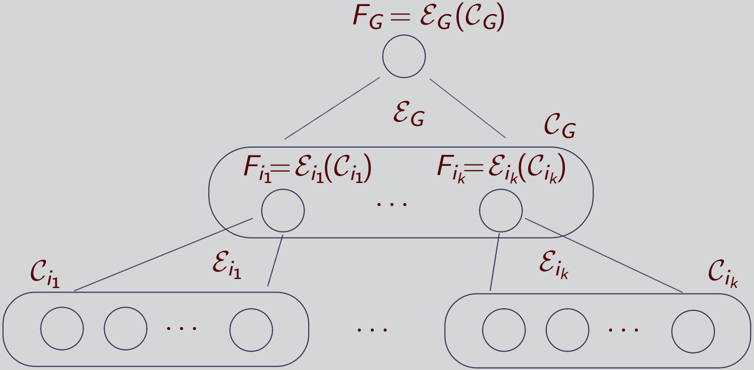
- Defines a schema of reasoning and dealing with conflicting and missing information.
- An *epistemic profile* is any function $\mathcal{E} : \mathsf{Fin}(\mathbb{C}) \longrightarrow \mathsf{Fin}(\mathbb{C})$ ($\mathcal{E}$: belief base $\longrightarrow$ belief base).

Belief Structures

A *belief structure over an epistemic profile* $\mathcal{E}$:
$\mathcal{B}^{\mathcal{E}} = \langle \mathcal{C}, F \rangle$, where:

- $\mathcal{C} \subseteq \mathbb{C}$ is a nonempty belief base of *constituents*;
- $F \stackrel{\text{def}}{=} \mathcal{E}(\mathcal{C})$ is the belief base of *consequents* of $\mathcal{B}^{\mathcal{E}}$.

Individual and Group Belief Structures [DKS]



Uniformity of Complex Beliefs

Individual vs Group Level

- Conceptual compatibility between individuals and groups.
- Consequents of group members become constituents at the group level.
- Constituents are further transformed into group consequents via groups' epistemic profiles.

Uniformity

The same uniform approach applies to groups of groups of agents or to mixed groups of individuals and other complex topologies.

Completing Imperfect Beliefs

Agents and Groups

- Non-monotonic reasoning.
- Defeasible reasoning.
- Heuristic methods.
- Argumentation theory inspired methods.

Group Specific

Social procedures to establish group beliefs:

- public announcements;
- different voting methods;
- methods involving power relations (like authority).

Conflicting Information

Resolving Conflicts

- Conflicting information may be resolved on the individual or group level in a similar way.
- Disambiguation methods are highly context, application-dependent and individualized.

Some Strategies as to Timing

- "Killing inconsistency at the root": to solve them asap;
- "Living with inconsistency": postpone disambiguation to the last possible moment (or even forever);
- Solving inconsistency whenever relevant information appears.

Truth Values of Atoms, $\ell(.,.)$

Let w be a 3ı-world and v be a valuation (assigning constants to variables).

$$\ell(w,v) \stackrel{\text{def}}{=} \begin{cases} \boldsymbol{t} & \text{if } \ell(v) \in w \text{ and } (\neg\ell(v)) \notin w; \\ \boldsymbol{i} & \text{if } \ell(v) \in w \text{ and } (\neg\ell(v)) \in w; \\ \boldsymbol{u} & \text{if } \ell(v) \notin w \text{ and } (\neg\ell(v)) \notin w; \\ \boldsymbol{f} & \text{if } \ell(v) \notin w \text{ and } (\neg\ell(v)) \in w. \end{cases}$$

Truth Ordering: How ”true” a Given Proposition Is

$$\begin{array}{c} \boldsymbol{t} \\ | \\ \boldsymbol{i} \\ | \\ \boldsymbol{u} \\ | \\ \boldsymbol{f} \end{array}$$

$A \wedge B = \min\{A, B\}$

$A \vee B = \max\{A, B\}$

$\forall x A(x) = A(a_0) \wedge \ldots \wedge A(a_n) \wedge \ldots$

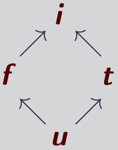
$\exists x A(x) = A(a_0) \vee \ldots \vee A(a_n) \vee \ldots$

($a_0, \ldots, a_n, \ldots$: all objects in the domain)

Belief Operators

$\mathrm{Bel}_\Delta(A)$: is formula A believed in wrt belief base Δ?

Semantics: via Information Ordering



$$\mathrm{Bel}_\Delta(A)(v) \stackrel{\mathrm{def}}{=} \operatorname{lub}_{w \in \Delta} \{A(w, v)\}.$$

Querying Belief Bases [DKS]

Fusion Operators

$\Phi^f_\Delta(A)$: what is the truth value of formula A in 3I-world $f(\Delta)$, where f is a mapping fusing the 3I-worlds of Δ into a single 3I-world?

Sample Fusion Operators

- $f(\Delta) \stackrel{\text{def}}{=} \bigcup_{w \in \Delta} w$.
- Voting.
- Authority-based.
- Argumentation-based.

Uniformity of Complex Beliefs

Individual Agents or Groups

Consequents		$\Delta \quad \rightsquigarrow \mathrm{Bel}_{\Delta}()$		
Epistemic profile	$\uparrow$	$\uparrow$	$\cdots$	$\uparrow$
	$\uparrow$	$\uparrow$	$\cdots$	$\uparrow$
	$\uparrow$	$\uparrow$	$\cdots$	$\uparrow$
Constituents	$\mathrm{Bel}_{\Delta_1}()$	$\Phi^{f_2}_{\Delta_2}()$	$\cdots$	$\mathrm{Bel}_{\Delta_k}()$
Infromation sources	$\underbrace{\Delta_1}_{\text{observations}}$	$\underbrace{\Delta_2}_{\text{other agent}}$	$\cdots$	$\underbrace{\Delta_k}_{\text{other group}}$

The Implementation Tool: 4ql [MS]

Foundations

- Possibly many, perhaps distributed information sources.
- Open World Assumption.
- Four logical values (***t***, ***f***, ***i***, ***u***).
- Modular structure.

Reasoning

- Unrestricted negation (in premises and conclusions of rules).
- Simple tools: rules, modules and external literals supporting (lightweight versions of) (Local) CWA, autoepistemic reasoning, default reasoning, defeasible reasoning, etc.

The Implementation Tool: 4ql

Semantics

- Well-supported model (all conclusions supported by reasoning grounded in facts).
- For every 4ql module there is exactly one WSM.

Complexity

- PTime complexity of computing models and queries.
- Captures all tractable queries.
- An experimental open source interpreter Inter4QL is available via 4ql.org.

Modules

Syntax

```
module name:
 domains: ...
 relations: ...
 rules: ...
 facts: ...
end.
```

Connectives

In the remaining slides ‘`,`’, ‘`;`’ and ‘`-`’ stand for conjunction, disjunction and negation, respectively.

Multisource Formulas

Use first-order formulas extended by expressions of the form:

- `m.F`;
- `m.F in T`,

where `m` is a module name, `F` is a formula (in extended language) and $\mathtt{T} \subseteq \{\boldsymbol{t}, \boldsymbol{i}, \boldsymbol{u}, \boldsymbol{f}\}$.

Cyclic references among modules not allowed

Querying Modules

Examples

- `persons.buys(C,P)`
  – select from module `persons` customers (`C`) and products they buy (`P`);
- `persons.buys(C,P) in {`***i***`,`***t***`}`
  – select from module `persons` customers and products where `buys(C,P)` is ***i*** or ***t***.
- `persons.buys(C,P), products.item(P,tv,low_quality)`
  – select from modules `persons` pairs $\langle$`C`, `P`$\rangle$ representing customers (`C`) buying products (`P`)
  claimed by module `products` to be low quality tvs.

The Implementation Tool: 4ql

Syntax of Rules

Rules:

`conclusion :– premises.`

where:

- `conclusion` is a (positive or negative) literal;
- `premises` are expressed by a multisource formula.

Facts

Rules with empty premises are called *facts*.

Empty premises evaluate to ***t***.

Semantics of Rules

- If `premises` evaluate to ***t*** then `conclusion` is in the set of beliefs (in the model).
- If `premises` evaluate to ***i*** then both `conclusion` and `-conclusion` are in the set of beliefs (in the model).

Russell's Paradox (1901)

Popular Formulation

```
 shaves(barber,X):- -shaves(X,X).
-shaves(barber,X):- shaves(X,X).
 savesMoney(X) :- shaves(X,X).
-savesMoney(X) :- -shaves(X,X).
 shaves(marc,marc).
-shaves(jack,jack).
 shaves(barber,barber).
```

Some Results

- `savesMoney(marc)` is true;
- `savesMoney(jack)` is false;
- `savesMoney(barber)` is inconsistent;
- `savesMoney(john)` is unknown.

Typical Sources of Nonmonotonicity

Generally, attempts to fill gaps in missing beliefs, e.g.,

- *efficient representation of (negative) information* (like Cwa, LCwa);
- *drawing rational conclusions from non-conclusive information* (e.g., circumscription, default logics);
- *drawing rational conclusions from the lack of knowledge* (e.g., autoepistemic reasoning);
- *resolving inconsistencies* (e.g., defeasible reasoning).

Local Closed World Assumption

Intuitively, one often wants to contextually close part of the world, not necessarily all relations in the database.

Example: Rules Locally Closing 'location'

```
 location(X,L,Now)  :-  nextTime(Now,Prev),
                        house(X),
                        cam.chngPos(X,Prev):- {u,f},
                        cam.location(X,L,Prev).
-location(X,L,Now)  :-  nextTime(Now,Prev),
                        movingCar(X),
                        cam.chngPos(X,Prev)∈{u,t},
                        cam.location(X,L,Prev).
```

Default Rules

Default rules have the form:

$$\textit{prerequisite} : \textit{justification} \vdash \textit{consequent},$$

with the intuitive meaning

"deduce *consequent* whenever *prerequisite* is true
and *justification* is consistent with the current beliefs".

Example: Expressing Default-like Rules

Default rule:

$$car(X) \wedge speed(X, high) : onRoad(X) \vdash onRoad(X)$$

captures similar intuitions as:

```
onRoad(X) :–   car(X),speed(X,high),
               cam.onRoad(X)∈{t,u}.
```

The Idea

- A typical pattern of autoepistemic reasoning:

  "If you *do not know* A, conclude $\neg A$."

- The rule stating:

  "If you do not know that you have a sister,
  conclude that you do not have a sister"

  can be expressed by a rule:

```
-have_sister(X):– family.have_sister(X) = u.
```

Example

Consider the following defeasible rules reflecting buyer's requirements as to apartments:

$$r1 : size(X, large) \Rightarrow acceptable(X)$$
$$r2 : \neg pets\_allowed(X) \Rightarrow \neg acceptable(X),$$

with priorities $r2 > r1$.

Example Continued

Assume module `m` contains rules:

```
 acceptable(X)  :-  size(X,large).
-acceptable(X)  :-  -pets_allowed(X).
```

Rules resolving inconsistencies according to the required priority:

```
 acceptable(X)  :-  m.acceptable(X) = t.
-acceptable(X)  :-  m.acceptable(X) in {f,i}.
```

The Link between 4ql and Belief Bases/Structures

Important

Every module can be identified with its well-supported model being a 3ı-world. Therefore:

- A module is a concise, uniform representation of 3ı-worlds.
- A set of modules is a concise representation of a belief base.
- Every epistemic profile computable in PTIME can be implemented using 4ql modules.

$4ql^{Bel}$ [BDKS]

Implementation tool supporting $\mathrm{Bel}()$, $\Phi()$ and belief shadowing, extending 4ql, will be available in 2018.

Final Example

The Group

The group consists of three agents: $\{p, n, e\}$, equipped with:

- p: a sensor platform for detecting air pollution;
- n: a sensor for measuring the noise level;
- e: a sensor platform for locating places.

The Task

Decide whether conditions in considered locations are healthy.

Final Example – Continued

Relations (Gathered by Agents p, n, e)

- C_p gathers beliefs about air pollution at given places.
  $P(X, Y)$ indicates the pollution level at place X,
  $Y \in \{low, moderate, high\}$;
- C_n gathers beliefs about noise level at given places.
  $N(X, Y)$ indicates the noise level at place X,
  $Y \in \{low, moderate, high\}$;
- C_e gathers information about the environment.
  $Cl(X, Y)$ indicates that place X is close to place Y.

Final Example – Continued

Constituents

$$C_p = \{P(b, high), P(c, high)\}$$
$$C_n = \{N(b, low), N(c, moderate)\}$$
$$C_e = \{Cl(a, b), Cl(b, a), Cl(a, c), Cl(c, a)\}$$

Remark

Nothing is known about pollution and noise levels in a place a.

Final Example – Continued

(A Part of) C_p's Epistemic Profile

A sample (default-like) rule:

```
P(X,moderate) :- P(X,high) in {f,i,u},
                 P(X,low) in {f,i,u},
                 Ce.Cl(X,Y),
                 P(Y,high).
    ...
```

Final Example – Continued

(A Part of) Group's Epistemic Profile

```
 S(X,healthy) :- Cn.N(X,moderate),
                 Cp.P(X,high) in {u,f}.
-S(X,healthy) :- Cp.P(X,moderate).
     ...
```

Final Example – Continued

Consequents

Based on constituents and its epistemic profile, the group derives an inconsistent consequent:

$$F = \{\neg S(a, healthy), S(a, healthy)\}.$$

Conclusions

- We differentiate agent's/group's characteristics via individual and group epistemic profiles.
- The framework:
  - suits real-world applications;
  - allows for natural handling of inconsistencies and gaps in beliefs by using paraconsistent and nonmonotonic reasoning;
  - ensures a uniform modeling of individual and group beliefs, where group is a generic concept consisting of individual agents, groups of agents, groups of groups of agents, etc.

Conclusions

- The architecture permits to avoid costly revisions of agents' beliefs when they join a group. This is especially important when paradigmatic agent interaction is considered.
- Cooperation, coordination and communication is naturally modeled by creating a group and forming group beliefs to achieve a common informational stance.

Conclusions

Tractability!

4ql guarantees tractability and is sufficient to implement all epistemic profiles and belief structures constructible in deterministic polynomial time.

Some Related Papers

Belief Structures and Epistemic Profiles

- B. Dunin-Kęplicz, A. Szałas:
  Epistemic Profiles and Belief Structures.
  KES-AMSTA, LNCS 7327, 360-369, 2012.
- B. Dunin-Kęplicz, A. Szałas: *Taming Complex Beliefs*.
  LNCS Trans. on Comp. Collect. Intelligence 11, LNCS 8065, 2013.
- B. Dunin-Kęplicz, A. Szałas:
  Paraconsistent Distributed Belief Fusion.
  6th Int. Symp. on Intelligent Distributed Computing, Studies in Computational Intelligence 446, 59-69, 2013.
- Ł. Białek, B. Dunin-Kęplicz, A. Szałas:
  Rule-based Reasoning with Belief Structures.
  ISMIS 2017, LNCS , LNCS10352, 229-239, 2017.

Some Related Papers

4QL (See Also `4ql.org`)

- J. Małuszyński, A. Szałas:
  Partiality and Inconsistency in Agents' Belief Bases.
  KES-AMSTA, Frontiers of AI and Applications 252, 3-17, 2013.
- A. Szałas: *How an Agent Might Think*.
  Logic Journal of IGPL 21(3), 515-535, 2013.
- J. Małuszyński, A. Szałas:
  Living with Inconsistency and Taming Nonmonotonicity.
  Datalog Reloaded, LNCS 6702, 384-398, 2011.
- J. Małuszyński, A. Szałas:
  Logical Foundations and Complexity of 4QL, a Query Language with Unrestricted Negation.
  Journal of Applied Non-Classical Logics 21(2), 211-232, 2011.

Some Related Papers

Belief Structures in Dialogues and Argumentation

- B. Dunin-Kęplicz, A. Strachocka:
  Tractable Inquiry in Information-Rich Environments.
  Proc. 24th IJCAI, 53–60, 2015.
- B. Dunin-Kęplicz, A. Strachocka:
  Multi-Party Persuasion: a Paraconsistent Approach.
  Fundamenta Informaticae 158(1-3): 1-39, 2018.
- B. Dunin-Kęplicz, A. Powała (Strachocka), A. Szałas:
  Variations on Jaśkowski's Discursive Logic.
  The Lvov-Warsaw School. Past and Present, 485-497, 2018.
- B. Dunin-Kęplicz, A. Strachocka, A. Szałas, R. Verbrugge:
  Perceiving Speech Acts under Incomplete and Inconsistent Information.
  KES AMSTA, Frontiers of AI and Applications 252, 255-264, 2013.